UG/ Sem III / MJC 4 04.11.2025

Reducible Linear Equations or (Bernoulli's equations) (contd.)

I Solve $x\sin\theta\, d\theta + (x^3 - 2x^2\cos\theta - \cos\theta)\,dx = 0.$

Soln

$\therefore x^3 dx = -x\sin\theta\, d\theta + (2x^2\cos\theta + \cos\theta)dx$

$\Rightarrow x^3 dx = -x\sin\theta\, d\theta + \cos\theta\, dx + 2x^2\cos\theta\, dx$

Dividing both sides by x^2, we get

$$\Rightarrow x\,dx = \frac{-x\sin\theta\, d\theta + \cos\theta\, dx}{x^2} + 2\cos\theta\, dx \quad \text{—(1)}$$

Put $\frac{\cos\theta}{x} = y \Rightarrow dy = \frac{-x\sin\theta\, d\theta + \cos\theta\, dx}{x^2}$ —(2)

using (2) in (1), we get

$\Rightarrow x\,dx = dy + 2.\,xy\,dx \quad [\because \cos\theta = xy]$

$\Rightarrow \frac{dy}{dx} + 2yx = x$ which is a linear eqn.

I.F. $= e^{\int 2x\,dx} = e^{x^2}$. Its soln. is given by

$y \times IF = \int Q \times IF\, dx \Rightarrow y e^{x^2} = \int x.\, e^{x^2} dx = \int e^{x^2}. d\left(\frac{x^2}{2}\right)$

$\Rightarrow y.e^{x^2} = \frac{1}{2} e^{x^2} + K \Rightarrow y = \frac{1}{2} + K e^{-x^2}$ But

$y = \frac{\cos\theta}{x} \Rightarrow \frac{\cos\theta}{x} = \frac{1}{2} + K e^{-x^2}$

$\Rightarrow 2\cos x = x + K x e^{-x^2}$